$\chi^2$

# 5.4 CHI^2 Test: Observed and Expected Frequencies

# $X^2$ Test for Independence

When you flip a coin 100 times, probability theory tells us that we can expect the same number of heads and tails:

Expected Frequencies

| Heads | Tails | Total |
|---|---|---|
| 50 | 50 | 100 |

Let's say you do the experiment with two different coins and get these results:

Observed Frequencies: Coin A

| Heads | Tails | Total |
|---|---|---|
| 48 | 52 | 100 |

Observed Frequencies: Coin B

| Heads | Tails | Total |
|---|---|---|
| 10 | 90 | 100 |

What might be a reasonable conclusion to draw?

Coin A: fair coin, close to expected outcomes

Coin B: unfair or weighted, possibly.

# $X^2$ Test for Independence

| | *Regular exercise* | *No regular exercise* | *sum* |
|---|---|---|---|
| *Male* | 110 | 106 | 216 |
| *Female* | 98 | 86 | 184 |
| *sum* | 208 | 192 | 400 |

What do you observe about the data?

- more males than females exercise.
- more people regularly exercise
- 400 people total.

What are some questions you might have based on this data?

- ages?
- Weights?
- Why more males than females?
- Where was sample taken?

# $X^2$ Test for Independence

The $\chi^2$ Test [Chi-Square Test] can help us decide whether or not two variables are independent. This means that one variable has no influence on the other variable.

Do you remember the definition of independence from probability?

If two events A and B are independent, then $P(A \cap B) = P(A) \cdot P(B)$.

The Chi-Square Test gives us a way to test independence by looking at the difference between observed frequencies and expected frequencies.

The expected frequency is calculated by $E = \frac{(row\ total)(column\ total)}{sample\ size}$.

# $X^2$ Test for Independence

Observed Frequency

| | *Regular exercise* | *No regular exercise* | *sum* |
|---|---|---|---|
| *Male* | 110 | 106 | 216 |
| *Female* | 98 | 86 | 184 |
| *sum* | 208 | 192 | 400 |

Expected Frequency if Independent

| | *Regular exercise* | *No regular exercise* | *sum* |
|---|---|---|---|
| *Male* | | | 216 |
| *Female* | | | 184 |
| *sum* | 208 | 192 | 400 |

What hypothesis could we test? Does gender affect exercise?

Null Hypothesis $H_0$: gender and regular exercise are independent

Alternative Hypothesis $H_1$:

gender and regular exercise are <u>not</u> independent

# $X^2$ Test for Independence

Observed Frequency

| | Regular exercise | No regular exercise | sum |
|---|---|---|---|
| Male | 110 | 106 | 216 |
| Female | 98 | 86 | 184 |
| sum | 208 | 192 | 400 |

Expected Frequency if Independent

| | Regular exercise | No regular exercise | sum |
|---|---|---|---|
| Male | 112.3 | 103.7 | 216 |
| Female | 95.7 | 88.3 | 184 |
| sum | 208 | 192 | 400 |

If gender and exercise are independent, then how many people would we expect to see in each category in the table?

Male ∩ Regular exercise:

$$\frac{216(208)}{400} = 112.3$$

Male ∩ No regular exercise:

$$\frac{216(192)}{400} = 103.7$$

Female ∩ Regular exercise:

$$\frac{184(208)}{400} = 95.7$$

Female ∩ No regular exercise:

$$\frac{184(192)}{400} = 88.3$$

# $X^2$ Test for Independence

Observed Frequencies

| | | Puppy | | | |
|---|---|---|---|---|---|
| | | Heavy | Medium | Light | Totals |
| Dog | Heavy | 23 | 16 | 11 | 50 |
| | Medium | 10 | 20 | 16 | 46 |
| | Light | 8 | 15 | 22 | 45 |
| | Totals | 41 | 51 | 49 | 141 |

Expected Frequencies

| | | Puppy | | | |
|---|---|---|---|---|---|
| | | Heavy | Medium | Light | Totals |
| Dog | Heavy | 14.5 | 18.1 | 17.4 | 50 |
| | Medium | 13.4 | 16.6 | 16.0 | 46 |
| | Light | 13.1 | 16.3 | 15.6 | 45 |
| | Totals | 41 | 51 | 49 | 141 |

What hypothesis could we test? puppy & dog size

Null Hypothesis $H_0$: puppy size and dog size are independent

Alternative Hypothesis $H_1$:

puppy size and dog size are not independent

Homework: WS 5.4 1-5 all